# UltraGroth: Interactive Groth16

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**Dmytro Zakharov**
**Distributed Lab**

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github.com/rarimo/ultragroth

# Why we should care?

# Range Checks

***Problem***

Write a circuit that checks whether $x$ is a 128-bit integer.

Current R1CS (and, consequently, Circom's) approach is to conduct the following steps:

- Find bit decomposition of $x$ off-circuit: say, $x = \sum_{i=0}^{127} x_i 2^i$.
- Check that $x_i \in \{0, 1\}$: impose 128 constraints $x_i(1 - x_i) = 0$.

**Result:** 128 constraints per *128-bit range check*.

***Question***

Suppose one needs to conduct 10000 such range checks. How many constraints does one need to implement this?

Using quite unsophisticated math, $128 \times 10000 =$ **1.28 mln**.

# Better range checks

Using lookup checks, we can implement the same logic in just $\approx$ 100k constraints! Here is how.

**Assumption.** Assume we can check whether the given signal $s$ is the $w$-bit integer in *a single constraint*. But this requires a *one-time* cost of $2^w$ constraints. How does it help us?

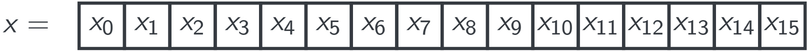
Suppose we use $w := 16$. Then, our algorithm proceeds as follows:

- We pay $2^{16} \approx 65.5$k for a one-time commitment.
- We find $w$-width decomposition of $x$: say, $x = \sum_{i=0}^{7} x_i 2^{wi}$.
- We check whether $x_i$ is a 16-bit integer. Since we have 8 chunks, this costs 8 constraints.

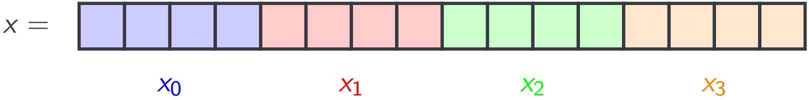
**Result:** We pay 65.5k constraints once and then every 128-bit range checks costs only 8 constraints instead of 128!

# Illustration

Let us illustrate this visually for a 16-bit range check over $x$!




16 constraints




4 constraints + one-time $2^4$ commitment

**Example:** 10000 such range checks would cost $16 \times 10000 = 160\text{k}$ constraints for a regular R1CS while $2^4 + 4 \times 10000 \approx 40\text{k}$ constraints over ZK system with lookups.

# Applications

- Wrappings of non-native ZKP verifications: e.g., zk-STARKs, sumcheck-based approaches.
- Non-native field arithmetic: e.g., optimized ECDSA verification for Rarimo passport verification.
- And surely, zero-knowledge Machine Learning — Bionetta.

| Framework | Metric | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | ResNet | MobileNet |
|---|---|---|---|---|---|---|---|---|---|
| **Bionetta (UltraGroth)** | Constraints # | 68.4K | 66.7K | 106.8K | 126.8K | 108.4K | 187.7K | 1.03M | 2.50M |
| | Proof Size (KB) | 1.20 | 1.20 | 1.20 | 1.20 | 1.20 | 1.20 | 1.20 | 1.20 |
| | PK (MB) | 48.40 | 50.60 | 80.60 | 106.30 | 81.90 | 156.20 | 0.95GB | 1.90GB |
| | VK (KB) | 3.78 | 3.79 | 3.78 | 3.78 | 3.78 | 3.78 | 4.05 | 4.20 |
| | Prove (s) | 0.57 | 0.73 | 0.74 | 1.08 | 0.89 | 1.79 | 6.27 | 15.22 |
| | Verify (s) | 0.006 | 0.005 | 0.005 | 0.006 | 0.006 | 0.005 | 0.006 | 0.006 |
| **Bionetta (Groth16)** | Constraints # | 29.0K | 5.9K | 522.4K | 779.4K | 543.0K | 1.56M | 12.01M | 31.78M |
| | Proof Size (KB) | 0.81 | 0.81 | 0.81 | 0.81 | 0.81 | 0.81 | 0.81 | 0.81 |
| | PK (MB) | 21.30 | 10.20 | 396.20 | 560.20 | 409.30 | 1.2GB | ≈9.0GB | ≈23.8GB |
| | VK (KB) | 3.65 | 3.65 | 3.65 | 3.65 | 3.65 | 3.65 | ≈4.0 | ≈4.0 |
| | Prove (s) | 0.12 | 0.27 | 2.19 | 2.20 | 2.22 | 4.72 | ≈180 | ≈480 |
| | Verify (s) | 0.006 | 0.006 | 0.006 | 0.006 | 0.006 | 0.005 | ≈0.005 | ≈0.006 |

Up to x12.7 boost in # of constraints!

# How to actually implement?

**Surprising result**: if the circuit consists of $L$ range-checks, each costing $b$ constraints, using lookup protocol, you can reduce $\mathcal{O}(n)$ constraints ($n = Lb$) down to $\mathcal{O}(n/\log n)$.

**Key question:** *how do we implement it* ***in Groth16?*** Since PlonK and SumCheck already have them! (see plookup+logup).

**Theorem (Some stuff from ZKDL Camp)**

*The inclusion check* $\{z_i\}_{i\in[n]} \subseteq \{t_i\}_{i\in[v]}$ *is satisfied if and only if there exists the set of multiplicities* $\{\mu_i\}_{i\in[v]}$ *where* $\mu_i = \#\{j \in [n] : z_j = t_i\}$ *such that for* $\gamma \xleftarrow{\$} \mathbb{F}$*:*

$$\sum_{i\in[n]} \frac{1}{\gamma + z_i} = \sum_{i\in[v]} \frac{\mu_i}{\gamma + t_i}$$

*High-level idea:* We can: (1) compute $\{\mu_i\}_{i\in[v]}$ off-circuit, (2) write circuit in $n + 2v$ constraints, given $\gamma$ signal is passed randomly.

# Circom-like Implementation

```
signal input t[M];          // The lookup table
signal random input gamma; // Random challenge value
signal input z[N];          // The array of values to check

var sum_z, sum_t = 0;
for (var i = 0; i < N; i++) {
    inv_z[i] <== 1 / (z[i] + gamma);
    sum_z += inv_z[i]; // Compute the left-hand side
}

for (var j = 0; j < M; j++) {
    mu[j] <-- 0; // Compute the multiplicities off-circuit
    for (var k = 0; k < N; k++) {
        mu[j] += (t[j] == z[k]);
    }
    inv_t[i] <== mu[j] / (t[j] + gamma);
    sum_t += int_v[i]; // Compute the right-hand side
}

sum_z === sum_t; // Check both sides are equal
```

# Problem

```
signal input t[M];           // The lookup table
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sum_z === sum_t; // Check both sides are equal
```

# UltraGroth Explained

# Some Historical Notes

- First paper on this problem is "MIRAGE: Succinct Arguments for Randomized Algorithms with Applications to Universal zk-SNARKs", published in 2020.
- Unaware of this protocol, in 2023 Lev Soukhanov published the post on **UltraGroth**, where he invented *multi-round* MIRAGE.
- Likely, unaware of Lev Soukhanov's blog, Alex Ozdemir, Evan Laufer, Dan Boneh published "Volatile and persistent memory for zkSNARKs via algebraic interactive proofs" paper in 2025.
- Well... Their construction, called MIRAGE+, is exactly an **UltraGroth**, published back in 2023.

*One important consequence*

The protocol is **safe**. It is sound and zero-knowledge! And it is now proven in **three** different independent papers.

# UltraGroth Performance

Now, let us recap the **Groth16** performance over the circuit of size $n$ and statement size $\ell$.

- **Prover work**: MSM of size $\mathcal{O}(n)$ over $\mathbb{G}_1$ and $\mathbb{G}_2$.
- **Proof size:** $2\mathbb{G}_1 + \mathbb{G}_2$.
- **Verifier work:** 3 pairings + $\mathcal{O}(\ell)$ $\mathbb{G}_1$ exps.

**UltraGroth** performance in turn:

- **Prover work**: MSM of size $\mathcal{O}(n/\log n)$ over $\mathbb{G}_1$ and $\mathbb{G}_2$.
- **Proof size:** $3\mathbb{G}_1 + \mathbb{G}_2$ (additional 64 bytes).
- **Verifier work:** 4 pairings + $\mathcal{O}(\ell)$ $\mathbb{G}_1$ exps + 1 hashing.

# UltraGroth Overall Idea

**Problem:** Compared to PlonK or SumCheck, *Groth16* itself is not derived from the interactive protocol (via Fiat-Shamir).

**Recap:** Proof in Groth16 consists of three points $g_1^{a(\tau)}$, $g_1^{c(\tau)}$, $g_2^{b(\tau)}$:

$$a(X) = \alpha + \sum_{i \in [n]} z_i \ell_i(X) + r\delta, \quad b(X) = \beta + \sum_{i \in [n]} z_i r_i(X) + s\delta,$$

$$c(X) = \delta^{-1} \left( \sum_{i \in \mathcal{I}_W} z_i \zeta_i(X) + h(X)t(X) \right) + a(X)s + b(X)r - rs\delta.$$

The **verification equation** is:

$$e(\pi_A, \pi_B) = e(g_1^{\alpha}, g_2^{\beta}) \cdot e(g_1^{i(\tau)}, g_2^{\gamma}) \cdot e(\pi_C, g_2^{\delta}).$$

for $\pi_A = g_1^{a(\tau)}$, $\pi_C = g_1^{c(\tau)}$, $\pi_B = g_2^{b(\tau)}$, $i(X)$ is a polynomial derived from the public statement.

# UltraGroth Overall Idea

- Do not touch $a(X)$ and $b(X)$.
- Split R1CS into two rounds: *round 0* computes the circuit without lookup check, *round 1* imposes lookup check.
- Split $c(X)$ into $c_0(X)$ and $c_1(X)$.
- $c_0(X)$ is derived from *round 0*'s witness.
- Form point $\pi_C^{\langle 0 \rangle} \leftarrow g_1^{c_0(\tau)}$ and sample randomness $\gamma \leftarrow \mathcal{H}(\pi_C^{\langle 0 \rangle})$.
- Compute witness for *round 1* using $\gamma$, form $c_1(X)$ and thus compute $\pi_C^{\langle 1 \rangle} \leftarrow g_1^{c_1(\tau)}$.
- Output proof as $\pi \leftarrow (\pi_A, \pi_C^{\langle 0 \rangle}, \pi_C^{\langle 1 \rangle}, \pi_B)$.

The **verification equation** is:

$$e(\pi_A, \pi_B) = e(g_1^{\alpha}, g_2^{\beta}) \cdot e(g_1^{i(\tau)}, g_2^{\gamma}) \cdot e(\pi_C^{\langle 0 \rangle}, g_2^{\delta_0}) \cdot e(\pi_C^{\langle 1 \rangle}, g_2^{\delta}).$$

**Note:** This construction can be easily generalized for $d > 1$ rounds.

# Our Contribution

- Implemented a single-round UltraGroth (essentially, a Mirage protocol). Credits to Artem Sdobnov, Vitalii Volovyk, Yevhenii Sekhin, and Illia Dovgopoly.
  - Forked `rapidsnark`.
  - Forked `snarkjs` for witness export/verify functions and smart-contract autogeneration.
  - Thanks to Ivan Lele, we even have a Swift SDK for that!
- Proved completeness, soundness, and zero-knowledge for general $d$-round UltraGroth. Formalized everything properly.
- Applied UltraGroth to Bionetta and obtained incredible results.

# Any Questions?

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